

THE UNEMPLOYMENT ANALYSIS USING A MEAN-REVERTING STOCHASTIC PROCESS

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Abstract : *in the next , a mean – reverting stochastic process having a discrete random walk component is performed to analyse the dynamic of the unemployment rate over the 2000 – 2005 period.*

The aim is to determine whether the unemployment rate will increase, decrease or stagnate

This class of processes belongs to one of the next two types :

$$S_{n+k} - S_n = f \left(\frac{S_n + \frac{1}{2}S_{n-1} + \frac{1}{3}S_{n-2} + \frac{1}{4}S_{n-3} + \dots + \frac{1}{h}S_{n-h+1}}{1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{h}} \right) + d \cdot e_k \text{ or}$$

$$S_{n+k} - S_n = f \left(\frac{S_n + \alpha \cdot S_{n-1} + \alpha^2 \cdot S_{n-2} + \alpha^3 \cdot S_{n-3} + \dots + \alpha^n \cdot S_0}{1 + \alpha + \alpha^2 + \dots + \alpha^n} \right) + d \cdot e_k$$

Here { S_n } is the process itself , f – is an appropriate function and { e_k } is an error variable.

We'll consider a stochastic process $\{S_n\}_{n \in \mathbb{N}}$ having the mean reverting property , verifying the next supplementary condition :

$$S_{n+k} - S_n = f(S^* - S_n) + \delta \cdot \epsilon_k \quad (1)$$

Here , ϵ_k – is a random walk process

S^* - is the global average of the process $\{S_n\}$

$f : \mathbb{R} \rightarrow (0 ; 1]$ is a continuous differentiable function,
such that $f(0) = 0 ; f'(0) = 0$.

For large enough values of “n” , we can interpret the relation (1) as

$$S_{n+k} - S_n \approx \delta \cdot \epsilon_k \quad (2)$$

if the condition

$$n \rightarrow \infty \text{ implies } |S^* - S_n| \rightarrow 0$$

is verified.

Let's now assume that ϵ_k – is a discrete, unitary random walk obtained as below.

Suppose a point M moves along the set Z of integer numbers :the corresponding time values are supposed to be natural numbers , $t \in \mathbb{N}$.

The next hypothesis are available :

(i): at the moment $t=0$, the point M lies in $n = 0$;

Let's denote by $A_t(\mathbf{n})$ the probabilistic event

$A_t(\mathbf{n}) =$ “ at the moment t , the point M lies in the position \mathbf{n} “

Then , from the moment t to the moment $t+1$, M jumps no more than one position , in any sense of the straight line.

Suppose that the next conditions are verified :

$$(ii): \mathbf{P}[A_{t+1}(n+1)|A_t(n)] = \mathbf{a}$$

$$(iii): \mathbf{P}[A_{t+1}(n-1)|A_t(n)] = \mathbf{b}$$

$$(iv): \mathbf{P}[A_{t+1}(n)|A_t(n)] = \mathbf{1-a-b}$$

with $a, b > 0$; $a + b \leq 1$.

Obviously , that :

- at the moment $t=0$, M lies in the position $n = 0$
- at the moment $t=1$, M can occupy only one of the position $n = -1$; $n = 0$ or $n = 1$ and the next relations are available :

$$\mathbf{P}[A_1(-1)] = \mathbf{b} ; \mathbf{P}[A_1(0)] = \mathbf{1-a-b} ; \mathbf{P}[A_1(1)] = \mathbf{a}$$

So , the random variable describing the position of the mobile point M at the moment $t = 1$ is the next one :

$$\mathbf{X}_1 = \begin{pmatrix} -1 & 0 & 1 \\ \mathbf{b} & \mathbf{1-a-b} & \mathbf{a} \end{pmatrix} ;$$

- at the moment $t=2$, M can occupy only one of the position $n = -2$; $n = -1$; $n = 0$; $n = 1$ or $n = 2$; the corresponding probabilities are the next

$$\begin{aligned} \mathbf{P}[A_2(-2)] &= \mathbf{b}^2 ; \\ \mathbf{P}[A_2(-1)] &= \mathbf{2 \cdot b \cdot (1-a-b)} ; \\ \mathbf{P}[A_2(0)] &= \mathbf{2 \cdot a \cdot b + (1-a-b)^2} ; \\ \mathbf{P}[A_2(1)] &= \mathbf{2 \cdot a \cdot (1-a-b)} ; \\ \mathbf{P}[A_2(2)] &= \mathbf{a}^2 , \end{aligned}$$

Then, the random variable describing the position of the mobile point M at the moment $t = 1$ is the next one :

$$\mathbf{X}_2 = \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ \mathbf{b}^2 & \mathbf{2 \cdot b \cdot (1-a-b)} & \mathbf{2 \cdot a \cdot b + (1-a-b)^2} & \mathbf{2 \cdot a \cdot (1-a-b)} & \mathbf{a}^2 \end{pmatrix}$$

;

Generally , the random variable \mathbf{X}_n of the positions at the moment $\mathbf{t = n}$ has the next properties :

- the arguments of \mathbf{X}_n are $\{ -n ; -n+1 ; -n+2 ; \dots ; -1 ; 0 ; 1 ; \dots ; n-1 ; n \}$
- the additive relation $\mathbf{X}_n = \mathbf{X}_1 + \mathbf{X}_1 + \dots + \mathbf{X}_1$ (n - terms) is verified

Much more, the next decomposition is available

$$\mathbf{X}_{n+k} = \mathbf{X}_n + \mathbf{X}_k , (\forall) \mathbf{n, k \in \mathbf{N}^*} .$$

As an example, we have $\mathbf{X}_3 = \mathbf{X}_1 + \mathbf{X}_2$, so

$$\begin{aligned}
P [X_3 = - 3] &= b^3 ; \\
P [X_3 = - 2] &= 3 \cdot b^2 \cdot (1 - a - b) ; \\
P [X_3 = - 1] &= 3 \cdot a \cdot b^2 + 3 \cdot b \cdot (1 - a - b)^2 ; \\
P [X_3 = 0] &= 6 \cdot a \cdot b \cdot (1 - a - b) + (1 - a - b)^3 ; \\
P [X_3 = 1] &= 3 \cdot a^2 \cdot b + 3 \cdot a \cdot (1 - a - b)^2 ; \\
P [X_3 = 2] &= 3 \cdot a^2 \cdot (1 - a - b) ; \\
P [X_3 = 3] &= a^3 .
\end{aligned}$$

Taking these into account, the relation (2) becomes :

$$S_{n+k} - S_n \approx \delta \cdot X_k \quad (3)$$

or

$$S_{n+1} - S_n \approx \delta \cdot X_1 \Leftrightarrow \frac{S_{n+1} - S_n}{\delta} \approx X_1 \quad (4)$$

As a consequence, for

$$|S_{n+1} - S_n| \leq \delta \Leftrightarrow -\delta \leq S_{n+1} - S_n \leq \delta \Leftrightarrow -1 \leq \frac{S_{n+1} - S_n}{\delta} \leq 1$$

in an obvious way , we get

$$\delta = \max_n \text{abs} \{ S_{n+1} - S_n \} \quad (5)$$

In the next , we'll deal with the next trouble : the variable $\frac{S_{n+1} - S_n}{\delta}$ covers all the interval $[-1 ; 1]$, not

only the arguments $\{-1 ; 0 ; 1\}$ of X_1 .

To make this work , a fuzzification technique will be applied .

$$\text{- let } \mathbf{X} = \begin{pmatrix} \mathbf{x} \\ \mathbf{f}(\mathbf{x}) \end{pmatrix}, \mathbf{x} \in [\mathbf{a}; \mathbf{b}] \text{ be a random variable with the density function}$$

$$\mathbf{f} : [\mathbf{a}; \mathbf{b}] \rightarrow [0 ; 1] .$$

Let $\Psi : [-1; 1] \rightarrow \{-1; 0; 1\}$ be the function given by

$$\Psi(p) = \begin{cases} -1, & \text{if } -1 \leq p < -\frac{1}{3} \\ 0, & \text{if } -\frac{1}{3} \leq p \leq \frac{1}{3} \\ 1, & \text{if } \frac{1}{3} < p \leq 1 \end{cases}$$

The discrete random variable $\hat{\mathbf{X}}$ which corresponds to \mathbf{X} will then be

$$\hat{\mathbf{X}} = \Psi \left[\frac{2 \cdot \mathbf{X} - (\mathbf{a} + \mathbf{b})}{\mathbf{b} - \mathbf{a}} \right] \quad (6)$$

Then ,

$$\hat{\mathbf{X}} = \begin{pmatrix} -1 & 0 & 1 \\ \alpha & \beta & \gamma \end{pmatrix} ,$$

with

$$\alpha = P\left[X < \frac{2 \cdot a + b}{3}\right]; \beta = P\left[\frac{2 \cdot a + b}{3} \leq X \leq \frac{a + 2 \cdot b}{3}\right]; \gamma = P\left[X > \frac{a + 2 \cdot b}{3}\right].$$

Observe now that, for $p \in \{-1; 0; 1\}$ we have $\Psi(p) = p$, so for the variable X_1 , the relation $\Psi(X_1) = X_1$ will be true.

Finally, from (4) we derive that

$$\frac{S_{n+1} - S_n}{\delta} \approx X_1 \Rightarrow \Psi\left(\frac{S_{n+1} - S_n}{\delta}\right) \approx X_1 \quad (5)$$

As an example, let's consider the next cross-section for the unemployment in the process $\{S_n\}_{n=0,5}$ representing the unemployment in ROMANIA, in the 2000 – 2005 period (<http://www.insse.ro/cms/files/pdf/en/cp3.pdf>):

n	0	1	2	3	4	5
s _n	10,5	8,8	8,4	7,4	6,3	5,9

We intend to estimate the corresponding X_1 repartition:

To do this, we'll compute first the estimated increases $\{\delta s_n\}$ of the process

n	0	1	2	3	4
$\delta s_n =$ $= s_{n+1} - s_n$	$10,5 - 8,8 =$ $= 1,7$	$8,8 - 8,4 =$ $= 0,41$	$8,4 - 7,4 =$ $= 1$	$7,4 - 6,3 =$ $= 1,1$	$6,3 - 5,9 =$ $= 0,4$

Then, $\delta = \max \text{abs}\{s_{n+1} - s_n\} = \max\{1,7; 0,41; 1; 1,1; 0,4\} = 1,7$.

By considering that the variable $S_{n+1} - S_n$ is distributed over the interval $[-\delta, +\delta] = [-1,7; 1,7]$, we obtain (see (6) above)

$$\begin{cases} a = -1,7 \\ b = +1,7 \end{cases} \Rightarrow \begin{cases} \frac{2 \cdot a + b}{3} = -0,57 \\ \frac{a + 2 \cdot b}{3} = +0,57 \end{cases}$$

In the list $\{\delta s_n = s_{n+1} - s_n\}$:

- we have no value lower than $\frac{2 \cdot a + b}{3} \Rightarrow P(X_1 = -1) = 0$

(the increase probability);

- three of the values belong to the interval $(-0,57; 0,57)$

$$\Rightarrow P(X_1 = 0) = \frac{3}{5}$$

(the stagnation probability);

- two of these values are larger than 0,57 $\Rightarrow P(X_1 = +1) = \frac{2}{5}$

(the decreasing probability).

Using the maximum likelihood method, we derive that

$$\mathbf{X}_1 = \begin{pmatrix} -1 & 0 & 1 \\ 0,17 & 0,58 & 0,25 \end{pmatrix}.$$

Finally, for the next period, the main probability is that, that the unemployment rate will stagnate.

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