THE UNEMPLOYMENT ANALYSIS USING A MEAN-REVERTING STOCHASTIC PROCESS

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Abstract : *in the next , a mean – reverting stochastic process having a discrete random walk component is performed to analyse the dynamic of the unemployment rate over the 2000 – 2005 period.*

The aim is to determine whether the unemployment rate will increase, decrease or stagnate

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This class of processes belongs to one of the next two types :

$$S_{n+k} - S_n = f\left(\frac{S_n + \frac{1}{2}S_{n-1} + \frac{1}{3}S_{n-2} + \frac{1}{4}S_{n-3} + \ldots + \frac{1}{h}S_{n-h+1}}{1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{h}}\right) + d \cdot e_k \text{ or}$$

$$S_{n+k} - S_n = f\left(\frac{S_n + \alpha \cdot S_{n-1} + \alpha^2 \cdot S_{n-2} + \alpha^3 \cdot S_{n-3} + \ldots + \alpha^n \cdot S_0}{1 + \alpha + \alpha^2 + \ldots + \alpha^n}\right) + d \cdot e_k$$

Here { Sn } is the process itself , f – is an appropriate function and { ek } is an error variable.

We'll consider a stochastic process $\{S_n\}_{n \in N}$ having the mean reverting property , verifying the next supplimentary condition :

$$S_{n+k} - S_n = f(S^* - S_n) + \delta \cdot \varepsilon_k \quad (1)$$

Here , ε_k – is a random walk process

S^* - is the global average of the process $\{S_n\}$

f : R → (0 ; 1] is a continuous differentiable function,

such that f(0) = 0 ; $f^{`}(0)=0$.

For large enough values of "n" , we can interpret the relation (1) as

$$S_{n+k} - S_n \approx \delta \cdot \varepsilon_k \qquad (2)$$

if the condition

$$n \to \infty \text{ implies } |S^* - S_n| \to 0$$

is verified.

Let's now assume that ε_k – is a discrete, unitary random walk obtained as below.

Suppose a point M moves along the set Z of integer numbers :the corresponding time values are supposed to be natural numbers , $t \in N$.

The next hypothesis are available :

(i): at the moment t=0 , the point M lies in n = 0 ;

Let's denote by $A_t(n)$ the probabilistic event

$A_t(n)$ = " at the moment **t ,** the point M lies in the position **n** "

Then , from the moment t to the moment t+1 , M jumps no more than one position , in any sense of the straight line.

Suppose that the next conditions are verified :

(ii): $\mathbf{P[A_{t+1}(n+1) | A_t(n)] = a}$

(iii): $\mathbf{P[A_{t+1}(n-1) | A_t(n)] = b}$

(iv): $\mathbf{P[A_{t+1}(n) | A_t(n)] = 1-a-b}$

with a , b > 0 ; a + b ≤ 1.

Obviously , that :

- at the moment t=0 , M lies in the position n = 0
- at the moment t=1 , M can occupy only one of the position n = -1 ; n = 0 or n =1 and the next relations are available :

$$\mathbf{P[A_1(-1)] = b \;;\; P[A_1(0)] = 1- a- b \;;\; P[A_1(1)] = a}$$

So , the random variable describing the position of the mobile point M at the moment t = 1 is the next one :

$$\mathbf{X_1 = \begin{pmatrix} -1 & 0 & 1 \\ b & 1-a-b & a \end{pmatrix}} \;;$$

- at the moment t=2 , M can occupy only one of the position n = -2 ; n = -1 ; n = 0 ; n =1 or n =2 ;the corresponding probabilities are the next

$P[A_2(-2)] = b^2$;
$P[A_2(-1)] = 2\cdot b\cdot(1- a- b)$;
$P[A_2(0)] = 2\cdot a\cdot b +(1- a- b)^2$;
$P[A_2(1)] = 2\cdot a\cdot(1- a- b)$;
$P[A_2(2)] = a^2$,

Then, the random variable describing the position of the mobile point M at the moment t = 1 is the next one :

$$\mathbf{X_2 = \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ b^2 & 2\cdot b\cdot(1-a-b) & 2\cdot a\cdot b+(1-a-b)^2 & 2\cdot a\cdot(1-a-b) & a^2 \end{pmatrix}}$$

;

Generally , the random variable $\mathbf{X_n}$ of the positions at the moment $\mathbf{t = n}$ has the next properties :

- the arguments of $\mathbf{X_n}$ are { - n ; - n+1 ;- n+2;…; -1 ;0 ;1 ;…; n-1 ; n }

- the additive relation $\mathbf{X_n = X_1+X_1+\ldots+X_1}$ (n – terms) is verified

Much more, the next decomposition is available

$$\mathbf{X_{n+k} = X_n + X_k \,,\; (\forall) n,k \in N^*} \;.$$

As an example, we have $\mathbf{X_3 = X_1+X_2}$, so

$P[X_3 = -3] = b^3$;
$P[X_3 = -2] = 3\cdot b^2\cdot(1-a-b)$;
$P[X_3 = -1] = 3\cdot a\cdot b^2 + 3\cdot b\cdot(1-a-b)^2$;
$P[X_3 = 0] = 6\cdot a\cdot b\cdot(1-a-b) + (1-a-b)^3$;
$P[X_3 = 1] = 3\cdot a^2\cdot b + 3\cdot a\cdot(1-a-b)^2$;
$P[X_3 = 2] = 3\cdot a^2\cdot(1-a-b)$;
$P[X_3 = 3] = a^3$.

Taking these into account,the relation (2) becomes :

$$\mathbf{S_{n+k} - S_n \approx \delta \cdot X_k} \qquad (3)$$

or

$$\mathbf{S_{n+1} - S_n \approx \delta \cdot X_1 \Leftrightarrow \frac{S_{n+1} - S_n}{\delta} \approx X_1} \qquad (4)$$

As a consequence, for

$$\mathbf{|S_{n+1} - S_n| \le \delta \Leftrightarrow -\delta \le S_{n+1} - S_n \le \delta \Leftrightarrow -1 \le \frac{S_{n+1} - S_n}{\delta} \le 1}$$

in an obvious way , we get

$$\mathbf{\delta = \max_n abs\{S_{n+1} - S_n\}} \quad (5)$$

In the next , we'll deal with the next trouble : the variable $\mathbf{\frac{S_{n+1} - S_n}{\delta}}$ covers all the interval **[-1 ; 1]** , not only the arguments { - 1; 0 ; 1 } of $\mathbf{X_1}$.

To make this work , a fuzzification technique will be applied .

- let $\mathbf{X = \begin{pmatrix} x \\ f(x) \end{pmatrix}, x \in [a;b]}$ be a random variable with the density function

$$\mathbf{f : [a; b] \to [0 ; 1] .}$$

Let $\mathbf{\Psi : [-1;1] \to \{-1;0;1\}}$ be the function given by

$$\mathbf{\Psi(p) = \begin{cases} -1 \text{ , if } & -1 \le p < -\frac{1}{3} \\ 0, \text{ if } & -\frac{1}{3} \le p \le \frac{1}{3} \\ 1 \text{ ,if } & \frac{1}{3} < p \le 1 \end{cases}}$$

The discrete random variable $\mathbf{\hat{X}}$ which corresponds to **X** will then be

$$\mathbf{\hat{X} = \Psi\left[\frac{2\cdot X - (a+b)}{b-a}\right]} \quad (6)$$

Then ,

$$\mathbf{\hat{X} = \begin{pmatrix} -1 & 0 & 1 \\ \alpha & \beta & \gamma \end{pmatrix}} ,$$

with

$$\alpha=P\left[X<\frac{2\cdot a+b}{3}\right];\beta=P\left[\frac{2\cdot a+b}{3}\leq X\leq\frac{a+2\cdot b}{3}\right];\gamma=P\left[X>\frac{a+2\cdot b}{3}\right].$$

Observe now that , for $p\in\{-1;0;1\}$ we have $\Psi(p)=p$,so for the variable X_1 , the relation $\Psi(X_1)= X_1$ will be true.

Finally , from (4) we derive that

$$\frac{S_{n+1}-S_n}{\delta}\approx X_1 \Rightarrow \Psi\left(\frac{S_{n+1}-S_n}{\delta}\right)\approx X_1 \quad (5)$$

As an example, let's consider the next cross- section for the unemployment in the process $\{ S_n\}_{n=0,5}$ representing the unemployment in ROMANIA ,in the 2000 – 2005 period (http://www.insse.ro/cms/files/pdf/en/cp3.pdf):

n	0	1	2	3	4	5
s_n	10,5	8,8	8,4	7,4	6,3	5,9

We intend to estimate the corresponding X_1 repartition :

<u>To do this ,</u> we'll compute first the estimated increases $\{ \delta s_n \}$ of the process

n	0	1	2	3	4
$\delta s_n =$ $= s_{n+1} - s_n$	10,5 – 8,8 = =1,7	8,8 – 8,4 = = 0,41	8,4 – 7,4 = = 1	7,4 – 6,3 = = 1,1	6,3 -5,9 = = 0,4

Then , $\delta=\max abs\{s_{n+1}-s_n\}=\max\{1{,}7;\ 0{,}41;\ 1;1{,}1;0{,}4\}=1{,}7$.

By considering that the variable $S_{n+1} – S_n$ is distributed over the interval $[- \delta , +\delta] = [- 1,7 ; 1,7]$, we obtain (see (6) above)

$$\begin{cases} a=-1{,}7 \\ b=+1{,}7 \end{cases} \Rightarrow \begin{cases} \dfrac{2\cdot a+b}{3}=-0{,}57 \\ \dfrac{a+2\cdot b}{3}=+0{,}57 \end{cases}$$

In the list $\{ \delta s_n = s_{n+1} - s_n \}$:

- we have no value lower than $\dfrac{2\cdot a+b}{3} \Rightarrow P(X_1=-1)=0$

  (the increase probability) ;

- three of the values belong to the interval (- 0,57 ; 0,57)

  $\Rightarrow P(X_1=0)=\dfrac{3}{5}$

  (the stagnation probability) ;

- two of these values are larger than 0,57 $\Rightarrow P(X_1=+1)=\dfrac{2}{5}$

  (the decreasing probability) .

Using the maximum likelyhood method,we derive that

$$\mathbf{X_1} = \begin{pmatrix} \mathbf{-1} & \mathbf{0} & \mathbf{1} \\ \mathbf{0{,}17} & \mathbf{0{,}58} & \mathbf{0{,}25} \end{pmatrix}.$$

Finally , for the next period, the main probability is that , that the unemployment rate will stagnate.

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