ABOUT THE INSURANCE MARKET EQUILIBRIUM : AN EXAMPLE

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Some aspects of an insurance model are analysed.
The insurance market equilibrium being either separating or pooling, some linear models can be used to promote more efficient allocation of consumption
We deal with a private market ,under the hypothesis that the households can save in a bequeathable asset as well .
Keywords : self-insurance , loss probability , utility function , critical set
*(*JEL: C , G*)*

In the case (see: 1) of an insurance market with adverse selection , consumers will not enjoy an effective insurance.
In the case of an insurance market with two types of customers, not having the same loss probability , Rotschild and Stiglitz shown that the one having the low loss probability will,in the case of the separating equilibrium , not even the net incomes.
This is the reason why they are looking for other ways to equilibrate their consumption.
The self-insurance option is taken into account .
Suppose that in a market there exist two different types of householders : T_1 and T_2 .
the next are known :

the type	the weight in the market	the loss probability
$\mathbf{T_1}$	$\boldsymbol{\pi_1}$	$\mathbf{p_1}$
$\mathbf{T_2}$	$\boldsymbol{\pi_2}$	$\mathbf{p_2}$

The pre-insurance wealth is $\eta_1 = m$; the possible loss is = k , the same for the two types : so,the final wealth will be $\eta_2 = m - k$.

Consider a utility function U , strictly increasing,stirctly concave , differentiable of order 2 (for example, U(x) = x /(1+x)).

For a consumption plan $\boldsymbol{c = (c_1 , c_2)}$, the utility repartitions are:

- for the householder T_1 : $\mathbf{U_1(c)=\begin{pmatrix} U(c_1) & U(c_2) \\ 1-p_1 & p_1 \end{pmatrix}}$; the average utility is given by

$$\mathbf{\overline{U_1(c)}=(1-p_1)\cdot U(c_1)+p_1\cdot U(c_2)}$$

- for the householder T_2 : $\mathbf{U_2(c)=\begin{pmatrix} U(c_1) & U(c_2) \\ 1-p_2 & p_2 \end{pmatrix}}$; the average utility is given by

$$\mathbf{\overline{U_2(c)}=(1-p_2)\cdot U(c_1)+p_2\cdot U(c_2)}$$

By signing an insurance contract $\boldsymbol{x}$, the pre-insurance wealth $\boldsymbol{\eta = (\eta_1 ;\eta_2)}$ turns to the post-insurance wealth $\boldsymbol{x = (x_1 ; x_2)}$ and the residual wealth becomes = $\boldsymbol{\eta - x}$.

The insurance contract is actuarially reliable for T_i if the average of $\boldsymbol{\eta} - \boldsymbol{x}$ is zero ,

i.e. :

$$(1-\mathbf{p_i})\cdot(\eta_1-\mathbf{x}_1)+\mathbf{p_i}\cdot(\eta_2-\mathbf{x}_2)=0$$

The actuarial reliability for the pair of householders is then given by the condition

$$\pi_1\cdot\overline{\mathbf{U}_1(\mathbf{c})}+\pi_2\cdot\overline{\mathbf{U}_2(\mathbf{c})}=0$$

Let's discuss now about a self-insurance option and his consequences :
a self-insurance option consists in giving up ***e*** unis of income in the good state , and increasing the income in the bad state by $\lambda\cdot e$.
So, the contract $\boldsymbol{x} = (x_1 ; x_2)$ becomes $(x_1 - e ; x_2 + \lambda\cdot e)$.

To provide an example, we-ll use the utility function

$$\mathbf{U(x)=ATANH}\left(\frac{\mathbf{x}}{\mathbf{x}+1}\right)$$

considered to be very suitable (2 ; 3) in studying equilibrium problems .

To find the optimal decizion for a householder, we have only to look for the maximum value of the average utility , i.e.:

$$\mathbf{F(e;\lambda)=(1-p)\cdot U(x_1-e)+p\cdot U(x_2+\lambda\cdot e)}.$$

The critical set for $\mathbf{F(e;\lambda)}$, denoted by $\mathbf{K(x_1,x_2,p)}$, is the interior of an ellipse in the $(\mathbf{e0\lambda})$ plane , taking into account that $\frac{\partial \mathbf{F}}{\partial \lambda}$ does not vanish in the first quadrant in $(\mathbf{e0\lambda})$.

In the case of the two customers $T_{1,2}$ as above , there deduce the next :

- if the two critical sets are disjoint , then insurance market equilibrium is separating
- if no , the insurance market equilibrium is pooling.

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