ADOPTION DECISION SUPPORT IN USE OR REPLACEMENT OF MEANS OF TRANSPORT USING OPTIMAL CONTROL THEORY

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Abstract

Current structures of the Romanian road system, fail, due to the multitude of bureaucratic regulations and rigid hierarchical structures, to meet the requirements of a dynamic market and its social effects. Processing new information generated by the new organizational structure, management using mathematical tools, is an immediate requirement to revisit traditional managerial concepts based on hierarchical structures and bureaucratic regulations. The latter, no longer cope with the conditions of transformation and reform to address the new objectives. They are always multiple and new or because in practice there is no settlement models applicable one to one, is that successful models developed in societies with market economies can not be transferred, rather than conditioning, because they come from socio-economic environment and culturally distinct. They have adapted to economic, regional, and socio-cultural space in each hand, if we are to successfully implement the results of the transformation process.

Mathematical methods used to study the maintenance and replacement of vehicles is the optimal control theory. To facilitate understanding of the mathematical model we recall some results from optimal control theory's emphasis on the principle of minimum Pontriaghin. [1]

Optimal control problem

In what follows, we note R^m that $m-$ dimensional Euclidean space, the scalar product is defined, marked $<\cdot,\cdot>$, and the Euclidean norm, denoted $//\cdot//$. We recall that if $x=(x_1,x_2,...,x_m)^T$, $y=(y_1,y_2,...,y_m)^T \in R^m$, then $<x,y>=x^T y = x_1y_1+x_2y_2+...+x_my_m$, again $//x//=\sqrt{<x,x>}=\sqrt{x_1^2+x_2^2+...+x_m^2}$.

Home optimal control problem which we deal below and the subject of several studies published so far, is the problem of optimal control type Bolz. Bolz optimal control problem requires a control system of the form:

(1.1) $x' = f(t,x(t),u(t)),\ x(t_0)=x_0, \quad x\in R^n,\ u\in R^r$

minimizing's functional of the cost, as defined by:

(1.2) $c(x(\cdot),u(\cdot)) = g(x(t_1)) + \int_0^{t_1} f_0(t,x(t),u(t))dt$

on the set of *admissible pairs* $(x(\cdot),u(\cdot))$. A pair $(x(\cdot),u(\cdot))$ is called *admissible* if $u(\cdot):[t_0,t_1]\to \bigcup \subset R^r$ the pair is an application for piecewise continuous control system (1.1) admits a solution $u(\cdot):[t_0,t_1]\to R^n$ with predetermined properties. We note P_a the set admissible pairs. If a couple $(x(\cdot),u(\cdot))\in P_a$ is allowed, then the command $u(\cdot)$ is called admissible, and the trajectory $x(\cdot)$ is called admissible. Note with μ_a the crowd and the crowd accepted orders Ω_a accepted trajectories. The set $\bigcup \subset R^r$ with the property that $u(t)\in\bigcup,\ (\forall)t\in[t_0,t_1]$ is called crowd control parameters. Finally, we assume that the paths to check the restriction phase.

(1.3) $h(x(t))\le 0, \quad (\forall t\in[t_0,t_1])$ and the additional condition $h(x_0)<0$. Also, we assume that the functions $g(x),\ f(t,x,u),\ f_0(t,x,u)$ are continuous with continuous partial derivatives with respect to the second argument, and $h(x)$ has second order partial derivatives continuous. The fact that the function $f(t,x,u)$ is continuous with continuous partial derivatives on $x\in R^m$ ensures that each order accepted, the control system (1.1) admits at least one solution (trajectory permitted). The general theory of differential equations and the conditions imposed on the functions $f(\cdot,\cdot,\cdot)$ and $u(\cdot)$ pathways that control system allowed (1.1) are, in general, absolutely continuous functions. In what follows, the *pair admitted optional* we mean any pair $(\tilde{x}(\cdot),\tilde{u}(\cdot))\in\Omega_a\times\mu_a$ for which

(1.4) $c(\tilde{x}(\cdot),\tilde{u}(\cdot))\le c(x(\cdot),u(\cdot))$

for any admissible pair is a pair $(x(\cdot),u(\cdot))\in\Omega_a\times\mu_a$. If $(\tilde{x}(\cdot),\tilde{u}(\cdot))$ is a optional pair, then $\tilde{x}(\cdot)$ is called the corresponding optimal trajectory corresponding to the optimal control $\tilde{u}(\cdot)$.

Specific case of the economic challenge of maintaining the service or replacement of vehicles

A transport firm acquires a certain date a means of transport. Profits from the operation of means of transport decreases over time due to outdated moral means of transport is subject. To maintain the transport capacity of the transportation means of transportation, the company supports a number of expenses (maintenance, repair), then when its use is no longer profitable company decide to re-purpose other means of transport (staff training, reducing transport route, transport of other types of products, etc..) or its sale. Present mathematical models that are aimed at determining the part of the way in which the expenditure is incurred to maintain the vehicle, and on the other hand, sell or change the timing of the work which it was intended. The mathematical model relates to a transportation company which belongs to as long as it is used in the same transport activity.

The data shown in the modeling problem is the following:

c_0 - Acquisition cost of the transportation

$x(t)$ - Residual value at the time of the transportation

$u(t)$- The cost of maintaining the means of transport operated until

$b(t)$ - Maintaining cost effectiveness until your means of transport (expressed appreciation suitable transport means for maintaining an expense equal to unity)

$a(t)$- Wear the means of transport at the time

p - Corresponding to a unit profit value of the transportation

δ - Rate actions

T - When the transporting vehicle is sold

It adopts an updated form factor $\alpha(t)=e^{-\delta t}$.

Among these data are assumed known $c_0, a(t), b(t), p$ *şi* δ.

And consider continuous functions $a(t)$ and $b(t)$, $a(t)$ is increasing and $b(t)$ decreasing. These assumptions are natural because, over time, increase $a(t)$ equipment wear and maintenance expenditure efficiency $b(t)$ decreases. Also, we assume $p>\delta$. Occupation costs of maintaining the vehicle $u(\cdot)$, admitting that it is piecewise continuous and is bounded, ie there is $M>0$ such that

(2.1) $0\le u(t)\le M,\quad (\forall)t\in[0,T]$

Contributing to the maintenance costs incurred until t the means of transport $\int_0^t a(s)u(s)ds$ is increasing the value and $\int_0^t a(s)ds$ represents the decrease in the equipment due to wear. Therefore, the residual value of transport equipment at a time is given by the formula:

(2.2) $x(t)=c_0+\int_0^t a(s)u(s)ds-\int_0^t a(s)ds$

Since $a(t)$ is continuous and $u(t)$ is piecewise continuous, from (2.2) and deduce properties of the Riemann integral that $x(t)$ is a differentiable function. Differentiating both members of the relation (2.2), we deduce that the function $x(t)$ satisfies the differential equation (process control).

(2.3) $x'(t)=-a(t)+b(t)u(t)$ with initial condition.

(2.4) $x(0)=c_0$

Note that the function in equation (2.2) can be written in the form of equivalent

(2.5) $x(t)=c_0+\int_0^t[-a(s)+b(s)u(s)]ds$

If you are made to maintain the maximum allowable expenses, ie when $u(t)=M, \quad t\in[0,T]$ the transport vehicle residual value is $x_M(t)=c_0+\int_0^t[-a(s)+Mb(s)]ds, \quad t\geq 0$.

We assume that the function $t\mapsto \int_0^t[-a(s)+Mb(s)]ds, \quad t\geq 0$ is decreasing. In other words, even if carried out to maintain the maximum allowable expenses, the means of transport decreases. The benefit obtained by means of transport at the time of resale $T>0$ is

(2.6) $c(x(\cdot),u(\cdot))=x(T)e^{-\delta T}+\int_0^T[px(t)-u(t)]e^{-\delta t}dt-c_0$

The first term in equation (2.6) represents the cost of the transportation sales update from baseline to purchase, and the second term is updated global surplus of income maintenance expenditure. I got such an optimal control problem where the command trajectory $x(t)$ is allowed trajectory and $u(t)$ accepted control. Therefore, this problem is to maximize cost's functional $c(x(\cdot),u(\cdot))=x(T)e^{-\delta T}+\int_0^T[px(t)-u(t)]e^{-\delta t}dt-c_0$ admitted on the set of $x(\cdot),u(\cdot)$ admissible pairs, where $u(\cdot):[0,T]\to\bigcup=[0,M]\subset R$ is piecewise continuous function, and $x(\cdot):[0,T]\to R$ is absolutely continuous solution of the control system.

$x'(t)=-a(t)+b(t)u(t), \quad x(0)=c_0$ phase restriction

$0\leq u(t)\leq M, \quad (\forall)t\in[0,T], \quad T\geq 0$.

As there is pretty easy, the problem is formulated as a problem with the data type Bolz: $t_0=0, t_1=T, g(x(T))=x(T)e^{-\delta T}-c_0$,

$f(x,x(t),u(t))=-a(t)+b(t)u(t), f_0(t,x(t),u(t))=[px(t)-u(t)]e^{-\delta t}, h(x)=0$ and control parameters set $\bigcup=[0,M]$.

Bolz main tool for solving optimal control problem is to *Pontriaghin's minimum principle.* To enunciate the fundamental result of optimal control theory requires some definitions. First, Bolz optimal control problem is associated with a function $H(t,x,\psi)$ called hamiltonianmul problem and is defined by

(3.1) $\widetilde{H}(t,x,\psi,u)=\langle\psi, f(t,x,u)\rangle+f_0(t,x,u)$, where is the solution of differential equation $\psi(\cdot):[t_0,t_1]\to R^r$.

(3.2) $\psi'(t)=-\frac{\partial\widetilde{H}}{\partial x}\left(t,x(t),\psi(t),u(t)+\lambda(t)\frac{\partial p}{\partial x}(t,x(t),u(t))\right)$ with final condition

(3.3) $\psi(t_1)=\alpha_0\frac{\partial g}{\partial x}(x(t_1))+\alpha_1\frac{\partial h}{\partial x}(x(t_1))$ where $\alpha_0\geq 0, \quad \alpha_1\geq 0$.

The function $\psi(t)$ is called *deputy variable* and

(3.4) $p(t,x,u)=\left\langle\frac{\partial h}{\partial x}(x), \quad f(t,x,u)\right\rangle$

Also, the function $\lambda(\cdot):[t_0,t_1]\to(-\infty,0]$ is monotone increasing and $\lambda(t_1)=0$ right continuous on $[t_0,t_1]$.

For Bolz optimal control problem formulated in Section 4.6.1., Pantriaghin's minimum principle is stated as follows:

Pontriaghin's minimum principle. Let $(\widetilde{x}(\cdot),\widetilde{u}(\cdot))$ be an optimal pair, and $\frac{\partial h}{\partial x}(\widetilde{x}(t))\neq 0$ if $h(\widetilde{x}(t))=0$. Then there are constants $\alpha_0\geq 0, \alpha_1\geq 0$ and increasing function $\lambda(\cdot):[t_0,t_1]\to(-\infty,0]$, right continuous, $\lambda(t_1)=0$ and $\alpha_0+\alpha_1+|\lambda(t_0)|\neq 0$ such that

a) $\min_{u\in\bigcup}\widetilde{H}(t,\widetilde{x}(t),\psi(t),u)-\lambda(t)p(t,\widetilde{x}(t),u)=\widetilde{H}(t,\widetilde{x}(t),\psi(t),\widetilde{u}(t))- \quad -\lambda(t)p(t,\widetilde{x}(t),\widetilde{u}(t)), \quad t\in[t_0,t_1]$;

b) $\alpha_1 h(\widetilde{x}(t_1))=0$;

c) $\lambda(t)$ constant on the intervals $h(\widetilde{x}(t)) < 0$.

The principle of minimum Pontriaghin above is adapted from the work of C. Vârsan [3] where they found the demonstration. For further details you may consult the work of Hestenes MR. [4]

Remark. If the problem Bolz optimal control phase contains several restrictions on the type (1.3), $h_i(x(t)) \le 0, \quad i = 1, 2, ..., q$ then the statement is amended Pontriaghin's minimum principle in the correct direction, containing several constants $\alpha_1 \ge 0, \alpha_2 \ge 0, 000, \alpha_q \ge 0$ and functions $\lambda_i(\cdot) : [t_0, t_1] \to (-\infty, 0], \quad i = 1, 2, ..., q$ with the properties b) and c). Also, we have

$p_i(t,x,u) = \left\langle \frac{\partial g_i}{\partial x}(x), f(t,x,u) \right\rangle, \quad i = 1, 2, ..., q$ instead of a single function $p(t,x,u)$.

So in this case, $\psi(t)$ the current deputy is the solution of differential equation $\Psi'(t) = -\frac{\partial \widetilde{H}}{\partial x}(t, x(t), \psi(t), u(t)) + \sum_{i=1}^{q} \lambda_i(t) \frac{\partial p_i}{\partial x}(t, x(t), u(t))$ with final condition $\Psi(t_1) = \alpha_0 \frac{\partial g}{\partial x}(x(t_1)) + \sum_{i=1}^{q} \alpha_i \frac{\partial h_i}{\partial x}(x(t_1))$.

Noting $\psi(t)$ deputy associated with variable control system (2.3), form the Hamiltonian $\widetilde{H}(t, x(t), \psi(t), u(t)) = \langle \psi(t), f(t, x(t), u(t)) \rangle + f_0(t, x(t), u(t))$,

where: $f(t, x(t), u(t)) = -a(t) + b(t)u(t)$ and $f_0(t, x(t), u(t)) = [px(t) - u(t)]e^{\delta t}$.

Therefore,

(4.1) $\widetilde{H}(t, x(t), \psi(t), u(t)) = \psi(t)[-a(t) + b(t)u(t)] + [px(t) - u(t)]e^{\delta t}$.

Since $h(x) = 0$, then from (3.4) that $p(t,x,u) = 0$. So, the differential equation (3.2), satisfying by a deputy variable becomes: $\Psi'(t) = -\frac{\partial \widetilde{H}(t, x(t), \psi(t), u(t))}{\partial x} = -pe^{-\delta t}$ ie

(4.2) $\Psi'(t) = -pe^{-\delta t}, \quad t \in [0, T]$ with final condition

(4.3). $\Psi(T) = e^{-\delta T}$

Integrating differential equation (4.2), we obtain $\Psi(t) = \int_T^t \left(-pe^{-\delta s}\right) ds + e^{-\delta T} = \int_t^T pe^{-\delta s} ds + e^{-\delta T} = -\frac{p}{\delta} e^{-\delta s} \Big|_t^T + e^{-\delta T} = -\frac{p}{\delta} e^{-\delta T} + + \frac{p}{\delta} e^{-\delta t} + e^{-\delta T}$

ie

(4.4) $\Psi(t) = \frac{p}{\delta} e^{-\delta t} + \frac{\delta - p}{\delta} e^{-\delta T}, \quad t \in [0, T]$.

Taking into account the relation (4.4), at the relation (4.1) we obtain

$$\begin{aligned}
\widetilde{H}(t, x(t), \psi(t), u(t)) &= \left(\frac{p}{\delta} e^{-\delta t} + \frac{\delta - p}{\delta} e^{-\delta T} \right) [-a(t) + b(t)u(t)] + \\
&+ [px(t) - u(t)]e^{\delta t} = e^{-\delta t} \left[\frac{p}{\delta} + \frac{\delta - p}{\delta} e^{-\delta(T-t)} \right] [-a(t) + b(t)u(t)] + \\
&+ [px(t) - u(t)]e^{\delta t} = e^{-\delta t} \left[\frac{p}{\delta} + \frac{\delta - p}{\delta} e^{-\delta(T-t)} \right] b(t)u(t) - e^{-\delta t} u(t) + \\
&+ px(t)e^{-\delta t} - a(t)e^{-\delta t} \left[\frac{p}{\delta} + \frac{\delta - p}{\delta} e^{-\delta(T-t)} \right] = \\
&= e^{-\delta t} \left[\left(\frac{p}{\delta} + \frac{\delta - p}{\delta} e^{-\delta(T-t)} \right) b(t) - 1 \right] u(t) - a(t)u(t) + px(t)e^{-\delta t}, \text{ so}
\end{aligned}$$

(4.5) $\widetilde{H}(t,x(t),\psi(t),u(t)) = e^{-\delta t}\left[\left(\frac{p}{\delta}+\frac{\delta-p}{\delta}e^{-\delta(T-t)}\right)b(t)-1\right]u(t)-a(t)u(t)+px(t)e^{-\delta t}$.

If we note $\omega(t)=\dfrac{\delta}{p-(p-\delta)e^{-\delta(T-t)}}$, $\quad t\in[0,T]$, then from (4.5) we get:

(4.6) $\widetilde{H}(t,x(t),\psi(t),u(t)) = e^{-\delta t}\left(\frac{b(t)}{\omega(t)}-1\right)u(t)-a(t)u(t)+px(t)e^{-\delta t}$.

Since, by definition $p>\delta$, we obviously $\omega(t)>0$ to anything $t\in[0,T]$. Moreover, since $\omega'(t)=\dfrac{\delta^2(p-\delta)e^{-\delta(T-t)}}{\left[p-(p-\delta)e^{-\delta(T-t)}\right]^2}>0$ that function $\omega(t)$ is strictly increasing on $[0,T]$. According to the principle of minimum Pontriaghin every time $t\geq 0$ optimal control $u(t)$ must maximize the Hamiltonian $\widetilde{H}(t,x(t),\psi(t)u(t))$ with the restriction
$0\leq u(t)\leq M, \quad (\forall)t\in[0,T]$.

Hamiltonian is a linear function $u(t)$, its maximum is achieved for:

(4.7) $u(t)=\begin{cases} M, & dacă\ b(t)>\omega(t) \\ 0, & dacă\ b(t)<\omega(t). \end{cases}$

If $b(t)=\omega(t)$ then $u(t)$ can take any value from $[0,M]$. Since the function $b(t)$ is decreasing and $\omega(t)$ is increasing function, we deduce that the equation $b(t)=\omega(t)$ has at most one solution $T_0\in(0,T)$, ie, there is at most one value $T_0\in(0,T)$ for which we have $b(T_0)=\omega(T_0)$.

From these considerations we deduce the following expression of the optimal control (chart 7):

(4.8) $u(t)=\begin{cases} M, & dacă\ t\in[0,T_0) \\ 0, & dacă\ t\in(T_0,T]. \end{cases}$

If $t=T_0$ then $u(t)$ can take any value in the interval $[0,M]$.

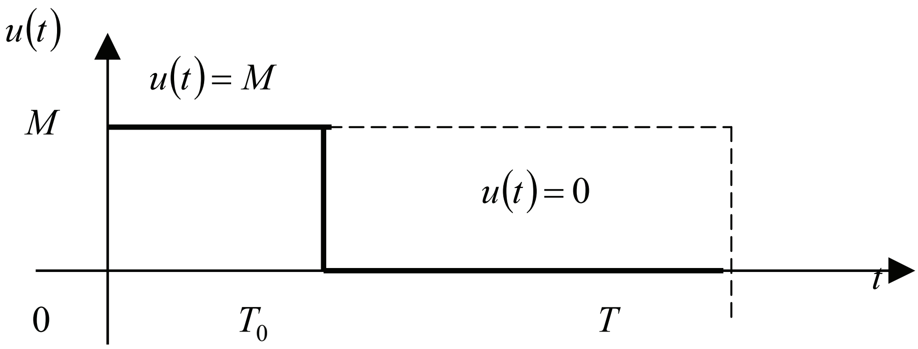

Chart 7. Optimal control

If $b(t)$ and $\omega(t)$ are not canceled on the interval $(0,T)$, then the optimal command is either $u(t)=0, \quad (\forall)t\in[0,T]$, either $u(t)=M, \quad (\forall)t\in[0,T]$. Moving to determine the optimal timing of the transport vehicle resale. For this we consider T variable to distinguish between policies that maintain optimal results under optimal control of (4.8). We note $u(t,T), \quad 0\leq t\leq T$ optimal maintenance costs when appropriate T.

The maximum benefit is the result:

(4.9) $y(T)=x(T)e^{-\delta T}+\int_0^T[px(t)-u(t,T)]e^{-\delta t}dt-c_0$ where

(4.10) $x(t)=c_0+\int_0^t[-a(t)+b(t)u(t,T)]dt$.

The optimal time to resell the machine will be solution of the equation: (4.11) $y'(T)=0$.

Differentiating in relation (4.9) to the variable T using the derivation rule of integrals which depend on parameters and taking into account that from (4.10) we obtain $x(t)=-a(t)+b(t)u(t)$, therefore $x'(T)=-a(T)+b(T)u(T)$, conclude that

(4.12) $$y'(T)=x'(T)e^{-\delta T}-\delta x(T)e^{-\delta T}+[px(T)-u(T,T)]e^{\delta T}+$$
$$+\int_0^T\frac{\partial}{\partial T}[px(t)-u(t,T)]e^{\delta t}dt=-\delta x(T)e^{-\delta T}++[-a(T)+b(T)u(T,T)]e^{\delta T}+$$
$$+[px(T)-u(T,T)]e^{\delta T}++\int_0^T\frac{\partial}{\partial T}[px(t)-u(t,T)]e^{\delta t}dt.$$

Function $u(t,T)$ being piecewise constant, it is easy to see that $\frac{\partial}{\partial T}[px(t)-u(t,T)]=0$.

According to Equation (4.8) we have, so from (4.12) we obtain

(4.13) $$y'(T)=-\delta x(T)e^{-\delta T}-a(T)e^{-\delta T}+px(t)e^{-\delta T}=[-a(T)+(p-\delta)x(T)]e^{\delta T}.$$

From (4.11) and (4.13) results $-a(T)+(p-\delta)x(T)=0$, where we get:

(4.14 $x(T)=\frac{a(T)}{p-\delta}$).

Equation (4.14) is the equation with the vendor that determines when the transport vehicle. Since the function $x(T)$ is decreasing and $a(T)$ is increasing, it follows that equation (4.14) has at most one solution (ie a unique solution, if it exists). In what follows, we consider an example for which we determine the optimal maintenance strategy and the optimal time to dispose of means of transport.

We consider the problem from section 4.1., Paragraph b) with the following data $c_0=10;\ a(t)=4;\ p=0{,}5;\ \delta=0{,}06;\ b(t)=\frac{1}{(t+1)^2};\ M=\frac{1}{3}$. In this case, the function $\omega(t)$ becomes $\omega(t)=\frac{\delta}{p-(p-\delta)e^{-\delta(T-t)}}=\frac{0{,}06}{0{,}5-(0{,}5-0{,}06)e^{-0{,}06(T_0-t)}}$.

Time optimal control T_0 of switching $u(t)$ is obtained from equality $b(T_0)=\omega(T_0)$, ie $\frac{1}{(T_0+1)^2}=\frac{0{,}06}{0{,}5-0{,}44e^{-0{,}06(T-T_0)}}$, where we get

(4.15) $25-22e^{-0{,}06(T-T_0)}=3(T_0+1)^2$.

Taking into account the relationship (2.5), optimal trajectory is given by $x(t)=c_0+\int_0^t[-a(s)+b(s)u(s)]ds$.

As $t<T_0$ for the optimal control is $u(t)=\frac{1}{3}$ (see equation (4.8)) gives:

$$x(t)=c_0+\int_0^t[-a(s)+Mb(s)]ds=10+\int_0^t\left[-4+\frac{1}{3(s+1)^2}\right]ds=\quad=10-4t-\frac{1}{3(t+1)}+\frac{1}{3}=\frac{31}{3}-4t-\frac{1}{3(t+1)}$$

And for $t>T_0$ the command optimal is $u(t)=0$, we get:

$$x(t)=c_0+\int_0^{T_0}[-a(s)+Mb(s)]ds+\int_{T_0}^t[-a(s)]ds=$$
$$=10+\int_0^{T_0}\left[-4+\frac{1}{3(s+1)^2}\right]ds+\int_{T_0}^t(-4)ds=10-4T_0-\frac{1}{3(T_0+1)}+\frac{1}{3}-4(t-T_0)=\frac{31}{3}-4t-\frac{1}{3(T_0+1)}$$

Therefore,

$$(4.16)\quad x(t)=\begin{cases}\frac{31}{3}-4t-\frac{1}{3(t+1)}, & dacă \quad t<T_0\\ \frac{31}{3}-4t-\frac{1}{3(T_0+1)}, & dacă \quad t>T_{0.}\end{cases}$$

Time to put the means of transport is the solution of equation (4.14), $x(T)=\dfrac{a(T)}{p-\delta}$ which for our problem becomes: $\dfrac{31}{3}-4T-\dfrac{1}{3(T_0+1)}=\dfrac{2}{0{,}44}$

where we get:

$$(4.17)\quad T=\frac{191}{132}-\frac{1}{12(T_0+1)}.$$

By replacing T in (4.15) with the value obtained in (4.17) we get:

$$(4.18)\quad 25-22e^{-0{,}06\left[\frac{191}{132}-\frac{1}{12(T_0+1)}-T_0\right]}-3(T_0+1)^2=0.$$

To determine the approximate solution of this equation using the electronic computer. Achieve $T_0\approx 0{,}22$. Substituting this value in (4.17) obtain $T=0{,}75$. Consequently, we obtain the following command to maintain optimal mode of transport:

$$u(t)=\begin{cases}\frac{1}{3}, & dacă \quad 0\le t<0{,}22\\ 0, & dacă \quad 0{,}22\le t\le 0{,}75.\end{cases}$$

Accordingly the strategy of keeping every time what means of transport is (cf. (4.16))

$$x(t)=\begin{cases}\frac{31}{3}-4t-\frac{1}{3(t+1)}, & dacă \quad 0\le t\le 0{,}22\\ -4t+9{,}06, & dacă \quad 0{,}22\le t\le 0{,}75.\end{cases}$$

In *conclusion,* we note that a situation faced by any transportation company, is that the continuation of vehicles, as long as their operation can be effective. Useful mathematical method to address this problem is the optimal control theory, which is based on the principle of minimum Pontriaghin. This method provides a general solution to address the problem of maintaining or replacing vehicles. Also, and this method is suitable for programming with the electronic computer. The analysis of case study concludes that the continuing operation of any means of transport over time deduced by mathematical methods, bringing losses to the transport company.

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