

# THE UNEMPLOYMENT ANALYSIS USING A MEAN-REVERTING STOCHASTIC PROCESS

**Reader Dan Petru Vasiliu ,Ph. D**

*The Titu Maiorescu University , Faculty of Economic Studies, Bucharest*

**Senior lect. Mihaela Iliescu Ph. D**

*The Titu Maiorescu University , Faculty of Economic Studies, Bucharest*

**Abstract :** *in the next , a mean – reverting stochastic process having a discrete random walk component is performed to analyse the dynamic of the unemployment rate over the 2000 – 2005 period.*

*The aim is to determine whether the unemployment rate will increase, decrease or stagnate*

This class of processes belongs to one of the next two types :

$$S_{n+k} - S_n = f \left( \frac{S_n + \frac{1}{2}S_{n-1} + \frac{1}{3}S_{n-2} + \frac{1}{4}S_{n-3} + \dots + \frac{1}{h}S_{n-h+1}}{1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{h}} \right) + d \cdot e_k \text{ or}$$

$$S_{n+k} - S_n = f \left( \frac{S_n + \alpha \cdot S_{n-1} + \alpha^2 \cdot S_{n-2} + \alpha^3 \cdot S_{n-3} + \dots + \alpha^n \cdot S_0}{1 + \alpha + \alpha^2 + \dots + \alpha^n} \right) + d \cdot e_k$$

Here {  $S_n$  } is the process itself ,  $f$  – is an appropriate function and {  $e_k$  } is an error variable.

We'll consider a stochastic process  $\{S_n\}_{n \in \mathbb{N}}$  having the mean reverting property , verifying the next supplementary condition :

$$S_{n+k} - S_n = f(S^* - S_n) + \delta \cdot \epsilon_k \quad (1)$$

Here ,  $\epsilon_k$  – is a random walk process

$S^*$  - is the global average of the process  $\{S_n\}$

$f : \mathbb{R} \rightarrow (0 ; 1]$  is a continuous differentiable function,  
such that  $f(0) = 0 ; f'(0) = 0$ .

For large enough values of “ $n$ ” , we can interpret the relation (1) as

$$S_{n+k} - S_n \approx \delta \cdot \epsilon_k \quad (2)$$

if the condition

$$n \rightarrow \infty \text{ implies } |S^* - S_n| \rightarrow 0$$

is verified.

Let's now assume that  $\epsilon_k$  – is a discrete, unitary random walk obtained as below.

Suppose a point  $M$  moves along the set  $Z$  of integer numbers :the corresponding time values are supposed to be natural numbers ,  $t \in \mathbb{N}$ .

The next hypothesis are available :

(i): at the moment  $t=0$  , the point  $M$  lies in  $n = 0$  ;

Let's denote by  $A_t(\mathbf{n})$  the probabilistic event

$A_t(\mathbf{n}) =$  “ at the moment  $t$  , the point  $M$  lies in the position  $\mathbf{n}$  “

Then , from the moment  $t$  to the moment  $t+1$  ,  $M$  jumps no more than one position , in any sense of the straight line.

Suppose that the next conditions are verified :

$$(ii): \mathbf{P[A_{t+1}(n+1)|A_t(n)] = a}$$

$$(iii): \mathbf{P[A_{t+1}(n-1)|A_t(n)] = b}$$

$$(iv): \mathbf{P[A_{t+1}(n)|A_t(n)] = 1-a-b}$$

with  $a, b > 0$  ;  $a + b \leq 1$ .

Obviously , that :

- at the moment  $t=0$  ,  $M$  lies in the position  $n = 0$
- at the moment  $t=1$  ,  $M$  can occupy only one of the position  $n = -1$  ;  $n = 0$  or  $n = 1$  and the next relations are available :

$$\mathbf{P[A_1(-1)] = b ; P[A_1(0)] = 1-a-b ; P[A_1(1)] = a}$$

So , the random variable describing the position of the mobile point  $M$  at the moment  $t = 1$  is the next one :

$$\mathbf{X_1 = \begin{pmatrix} -1 & 0 & 1 \\ b & 1-a-b & a \end{pmatrix} ;}$$

- at the moment  $t=2$  ,  $M$  can occupy only one of the position  $n = -2$  ;  $n = -1$  ;  $n = 0$  ;  $n = 1$  or  $n = 2$  ; the corresponding probabilities are the next

$$\begin{aligned} \mathbf{P[A_2(-2)]} &= b^2 ; \\ \mathbf{P[A_2(-1)]} &= 2 \cdot b \cdot (1-a-b) ; \\ \mathbf{P[A_2(0)]} &= 2 \cdot a \cdot b + (1-a-b)^2 ; \\ \mathbf{P[A_2(1)]} &= 2 \cdot a \cdot (1-a-b) ; \\ \mathbf{P[A_2(2)]} &= a^2 , \end{aligned}$$

Then, the random variable describing the position of the mobile point  $M$  at the moment  $t = 1$  is the next one :

$$\mathbf{X_2 = \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ b^2 & 2 \cdot b \cdot (1-a-b) & 2 \cdot a \cdot b + (1-a-b)^2 & 2 \cdot a \cdot (1-a-b) & a^2 \end{pmatrix}}$$

;

Generally , the random variable  $\mathbf{X_n}$  of the positions at the moment  $\mathbf{t = n}$  has the next properties :

- the arguments of  $\mathbf{X_n}$  are  $\{ -n ; -n+1 ; -n+2 ; \dots ; -1 ; 0 ; 1 ; \dots ; n-1 ; n \}$
- the additive relation  $\mathbf{X_n = X_1 + X_1 + \dots + X_1}$  ( $n$  - terms) is verified

Much more, the next decomposition is available

$$\mathbf{X_{n+k} = X_n + X_k , (\forall) n, k \in \mathbf{N}^* .}$$

As an example, we have  $\mathbf{X_3 = X_1 + X_2}$  , so

$$\begin{aligned}
P [ X_3 = - 3 ] &= b^3 ; \\
P [ X_3 = - 2 ] &= 3 \cdot b^2 \cdot (1 - a - b) ; \\
P [ X_3 = - 1 ] &= 3 \cdot a \cdot b^2 + 3 \cdot b \cdot (1 - a - b)^2 ; \\
P [ X_3 = 0 ] &= 6 \cdot a \cdot b \cdot (1 - a - b) + (1 - a - b)^3 ; \\
P [ X_3 = 1 ] &= 3 \cdot a^2 \cdot b + 3 \cdot a \cdot (1 - a - b)^2 ; \\
P [ X_3 = 2 ] &= 3 \cdot a^2 \cdot (1 - a - b) ; \\
P [ X_3 = 3 ] &= a^3 .
\end{aligned}$$

Taking these into account, the relation (2) becomes :

$$S_{n+k} - S_n \approx \delta \cdot X_k \quad (3)$$

or

$$S_{n+1} - S_n \approx \delta \cdot X_1 \Leftrightarrow \frac{S_{n+1} - S_n}{\delta} \approx X_1 \quad (4)$$

As a consequence, for

$$|S_{n+1} - S_n| \leq \delta \Leftrightarrow -\delta \leq S_{n+1} - S_n \leq \delta \Leftrightarrow -1 \leq \frac{S_{n+1} - S_n}{\delta} \leq 1$$

in an obvious way , we get

$$\delta = \max_n \text{abs} \{ S_{n+1} - S_n \} \quad (5)$$

In the next , we'll deal with the next trouble : the variable  $\frac{S_{n+1} - S_n}{\delta}$  covers all the interval  $[-1 ; 1]$  , not

only the arguments  $\{-1 ; 0 ; 1\}$  of  $X_1$  .

To make this work , a fuzzification technique will be applied .

$$- \text{ let } \mathbf{X} = \begin{pmatrix} \mathbf{x} \\ \mathbf{f}(\mathbf{x}) \end{pmatrix}, \mathbf{x} \in [\mathbf{a}; \mathbf{b}] \text{ be a random variable with the density function}$$

$$\mathbf{f} : [\mathbf{a}; \mathbf{b}] \rightarrow [0 ; 1] .$$

Let  $\Psi : [-1; 1] \rightarrow \{-1; 0; 1\}$  be the function given by

$$\Psi(p) = \begin{cases} -1, & \text{if } -1 \leq p < -\frac{1}{3} \\ 0, & \text{if } -\frac{1}{3} \leq p \leq \frac{1}{3} \\ 1, & \text{if } \frac{1}{3} < p \leq 1 \end{cases}$$

The discrete random variable  $\hat{\mathbf{X}}$  which corresponds to  $\mathbf{X}$  will then be

$$\hat{\mathbf{X}} = \Psi \left[ \frac{2 \cdot \mathbf{X} - (\mathbf{a} + \mathbf{b})}{\mathbf{b} - \mathbf{a}} \right] \quad (6)$$

Then ,

$$\hat{\mathbf{X}} = \begin{pmatrix} -1 & 0 & 1 \\ \alpha & \beta & \gamma \end{pmatrix} ,$$

with

$$\alpha = P\left[X < \frac{2 \cdot a + b}{3}\right]; \beta = P\left[\frac{2 \cdot a + b}{3} \leq X \leq \frac{a + 2 \cdot b}{3}\right]; \gamma = P\left[X > \frac{a + 2 \cdot b}{3}\right].$$

Observe now that, for  $p \in \{-1; 0; 1\}$  we have  $\Psi(p) = p$ , so for the variable  $X_1$ , the relation  $\Psi(X_1) = X_1$  will be true.

Finally, from (4) we derive that

$$\frac{S_{n+1} - S_n}{\delta} \approx X_1 \Rightarrow \Psi\left(\frac{S_{n+1} - S_n}{\delta}\right) \approx X_1 \quad (5)$$

As an example, let's consider the next cross-section for the unemployment in the process  $\{S_n\}_{n=0,5}$  representing the unemployment in ROMANIA, in the 2000 – 2005 period (<http://www.insse.ro/cms/files/pdf/en/cp3.pdf>):

n	0	1	2	3	4	5
s <sub>n</sub>	10,5	8,8	8,4	7,4	6,3	5,9

We intend to estimate the corresponding  $X_1$  repartition:

To do this, we'll compute first the estimated increases  $\{\delta s_n\}$  of the process

n	0	1	2	3	4
$\delta s_n =$ $= s_{n+1} - s_n$	$10,5 - 8,8 =$ $= 1,7$	$8,8 - 8,4 =$ $= 0,41$	$8,4 - 7,4 =$ $= 1$	$7,4 - 6,3 =$ $= 1,1$	$6,3 - 5,9 =$ $= 0,4$

Then,  $\delta = \max \text{abs}\{s_{n+1} - s_n\} = \max\{1,7; 0,41; 1; 1,1; 0,4\} = 1,7$ .

By considering that the variable  $S_{n+1} - S_n$  is distributed over the interval  $[-\delta, +\delta] = [-1,7; 1,7]$ , we obtain (see (6) above)

$$\begin{cases} a = -1,7 \\ b = +1,7 \end{cases} \Rightarrow \begin{cases} \frac{2 \cdot a + b}{3} = -0,57 \\ \frac{a + 2 \cdot b}{3} = +0,57 \end{cases}$$

In the list  $\{\delta s_n = s_{n+1} - s_n\}$ :

- we have no value lower than  $\frac{2 \cdot a + b}{3} \Rightarrow P(X_1 = -1) = 0$   
(the increase probability);
- three of the values belong to the interval  $(-0,57; 0,57)$   
 $\Rightarrow P(X_1 = 0) = \frac{3}{5}$   
(the stagnation probability);
- two of these values are larger than 0,57  $\Rightarrow P(X_1 = +1) = \frac{2}{5}$   
(the decreasing probability).

Using the maximum likelihood method, we derive that

$$\mathbf{X}_1 = \begin{pmatrix} -1 & 0 & 1 \\ 0,17 & 0,58 & 0,25 \end{pmatrix}.$$

Finally, for the next period, the main probability is that, that the unemployment rate will stagnate.

**BIBLIOGRAPHY :**

- 1: Sheldon M. Ross : "Stochastic processes" , John Wiley & Sons, NY 2007
- 2: Gropp ,W.; Lusk ,E. : " Dynamic Process Management " , Mathematics and Computer Science Division Argonne National Laboratory , 2008