Processing An Opinion Poll Using Fuzzy Techniques

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ABSTRACT: In this paper we deal with a multi criteria ranking problem, based on fuzzy input data : the purpose is to compare the effect of different metrics defined on the space of fuzzy numbers on the dynamics of the unique ranking.

As the main instrument, we use fuzzy synthesis to mediate different rankings ;

in this respect, the convolution product of fuzzy numbers occurs .A system of fuzzy weights is used, to enable the ordering criteria.

KEYWORDS: triangular fuzzy number ; similarity coefficient ; fuzzy multi-criteria decision making ; discrete normed fuzzy set

INTRODUCTION : In trying to make an available decision subject to imprecise and multicriteria situations, a decision maker is required to use a fuzzy multicriteria decision making method. *Fuzzy Multi-Criteria Decision Making* (MCDM) is based on fuzzy multi- attribute and multiobjective decision-making methodologies.

There are many MCDA / MCDM methods in use today , such as fuzzy AHP, fuzzy TOPSIS, interactive fuzzy multiobjective stochastic linear programming, fuzzy multiobjective dynamic programming, grey fuzzy multiobjective optimization, fuzzy multiobjective geometric programming, and more.

Unfortunately , different methods may yield different results for a given problem .

Choosing the best MCDA / MCDM method from a list of such methods is in turn a difficult multi-criteria decision making problem .This choice depends on the problem at hand and may dependent on which model the decision maker is most comfortable with .

In the case we'll use an opinion poll to determine – for example – a single final ranking for a set of objects, different viewpoints should be treated through a multicriteria procedure .

In such a case , the data can be treated as being fuzzy data : the fuzzy character appears to be generated by the diversity of functions of the object to be studied (see for example "Rule Based Fuzzy Classification using Squashing Functions " , by Zsolt Gera et al. , Szeged Univ. Hungary). Among the most used methods in this area , we mention the Analytic Hierarchy Process (AHP) , which will bw used as a model in developing our own version.

In a previous paper (1) we intend to classify some versions A_1, A_2 ,…, A_n with respect to a set of given criteria C_1, C_2 ,…, C_m.

Part of these criteria were " qualities" , their set being denoted by $I^+ = \{ C_1 ,\dots, C_k \}$;the others were " failures " ,denoted by $I^- = \{ C_{k+1} ,\dots, C_m \}$.

It's possible that one of this two sets I^+ or I^- be empty.

We'll start with a data matrix D , containing the ratings of the versions according to each criterion separately (see Fig. 1)

Fig. 1

	C_1	…	C_j	…	C_m
A_1	x_{11}	…	x_{1j}	…	x_{1m}
…	…	…	…	…	…
A_i	x_{i1}	…	x_{ij}	…	x_{im}
…	…	…	…	…	…
A_n	x_{n1}	…	x_{nj}	…	x_{nm}

The meaning of the quantities in Fig.1 is " the bigger , the better".

The set of criteria we deal are weighted criteria: the weight vector is denoted by $W = \{ w_j \}_{j=1,m}$, $w_j > 0$; $w_1 + ... + w_m = 1$.(see Fig.2).

Fig. 2

	C_1	…	C_j	…	C_m
A_1	x_{11}	…	x_{1j}	…	x_{1m}
…	…	…	…	…	…
A_i	x_{i1}	…	x_{ij}	…	x_{im}
…	…	…	…	…	…
A_n	x_{n1}	…	x_{nj}	…	x_{nm}
W	w_1	…	w_j	…	w_m

By following the method exposed in (1) , to determine the unique ranking of the versions A_1, A_2 ,…, A_n, there perform the next steps:

1: the normed matrix $C = \| r_{ij} \|$ is computed ,where (see (1))

$$\mathbf{r_{ij}} = \frac{\mathbf{x_{ij}}}{\sqrt{\sum_{j=1}^{m} \mathbf{x_{ij}}}} \quad (1)$$

2: there determines the normed weighted matrix $\hat{\mathbf{D}} = \| \mathbf{v_{ij}} \|_{i=1,n;\, j=1,m}$,with (see (2))

$$\mathbf{v_{ij}} = \mathbf{r_{ij}} \cdot \mathbf{w_j} \, (2)$$

-3:The connectivity vectors $\mathbf{D^+} = \{\mathbf{r_j^+}\}_{j=1,m}$, $\mathbf{D^-} = \{\mathbf{r_j^-}\}_{j=1,m}$ are computed by(see (3))

$$\mathbf{r_j^+} = \begin{cases} \mathbf{max_{i=1,n}\{ v_{ij} \}, daca\ C_j \in I^+} \\ \mathbf{min_{i=1,n}\{ v_{ij} \}, daca\ C_j \in I^-} \end{cases} \quad (3.a)$$

$$\mathbf{r_j^-} = \begin{cases} \mathbf{min_{i=1,n}\{ v_{ij} \}, daca\ C_j \in I^+} \\ \mathbf{max_{i=1,n}\{ v_{ij} \}, daca\ C_j \in I^-} \end{cases} \quad (3.b)$$

4:The Euclidean distances between the vectors D^+ , D^- and the rows of $\hat{\mathbf{D}} = \| \mathbf{v_{ij}} \|_{\mathbf{i=1,n;\, j=1,m}}$ are considered: this distance vectors are denoted by $Q^+ = (\mathbf{q_t^+})_{t=1,n}$,
$Q^- = (\mathbf{q_t^-})_{t=1,n}$

5: Finally , the rating vector S = { s_t }$_{t=1,n}$ for the versions A_1, A_2 ,…, A_n is given by (see (4))

$$\mathbf{s_t} = \frac{\mathbf{q_t^-}}{\mathbf{q_t^- + q_t^+}} \quad (4)$$

Consequently, a version A_t is even better, the more the value of s_t is larger.

To illustrate these , let's consider the next example : let the matrix D be as below (fig. 3)

Fig.3

		C_1	C_2	C_3	C_4	total
	A_1	3	2	1	1	7
	A_2	1	3	2	3	9
	A_3	2	1	3	2	8
	W	0,1	0,2	0,3	0,4	1

The intermediate matrix C , whose elements are given by $\mathbf{r_{ij}} = \frac{\mathbf{x_{ij}}}{\sqrt{\sum_{j=1}^{m} \mathbf{x_{ij}}}}$, becomes

(see Fig. 4)

	C_1	C_2	C_3	C_4
A_1	1,134	0,756	0,378	0,378

FIG. 4

A_2	0,333	1	0,667	1
A_3	0,707	0,354	1,061	0,707
W	0,1	0,2	0,3	0,4

The corresponding matrix $\tilde{\mathbf{D}}$ is presented below (Fig.5)

Fig. 5

	C_1	C_2	C_3	C_4
A_1	0,113	0,151	0,113	0,151
A_2	0,033	0,200	0,200	0,400
A_3	0,071	0,071	0,318	0,283

Suppose first that all the criteria are qualities :in this situation , the components of D^+ , D^- are given by (5)

$$\mathbf{r_j^+ = max_{i=1,n}\{ v_{ij} \}\ \ , pt. \forall\, j} \quad ; \qquad \mathbf{r_j^- = min_{i=1,n}\{ v_{ij} \}\ \ , pt. \forall\, j} \quad (5)$$

The table in Fig. 6 is thus obtained :

Fig. 6

D^+	0,113	0,200	0,318	0,400
D^-	0,033	0,071	0,113	0,151

The rating vector S , whose elements are given by $\mathbf{s_t = \frac{q_t^-}{q_t^- + q_t^+}}$,becomes

S =

0,258
0,674
0,580

Since $s_2 > s_3 > s_1$, we'll finally obtain the unique classification $\mathbf{C_2 \succ C_3 \succ C_1}$.

In the case all the criteria are failures , the vectors D^+ and D^- swap ,so , denoting by S_1 the new rating vector, we'll obviously have (6)

$S + S_1 =$

1
1
1

(6)

so (7)

$$S_1 = \begin{array}{|c|}\hline 0{,}742 \\ \hline 0{,}326 \\ \hline 0{,}420 \\ \hline \end{array} \qquad (7)$$

the new final classification being now $\mathbf{C_1} \succ \mathbf{C_3} \succ \mathbf{C_2}$

Conversely , if some of the criteria are qualities and the others are failures,say
$I^+ = \{ C_1 , C_2 \}$; $I^- = \{ C_3 , C_4 \}$, we'll have the next calculations:

$$\begin{aligned} &\mathbf{r_1^+ = max_{i=1,3}\{ v_{i1} \}; \; r_2^+ = max_{i=1,3}\{ v_{i2} \}; r_3^+ = min_{i=1,3}\{ v_{i3} \}; r_4^+ = min_{i=1,3}\{ v_{i4} \};} \\ &\mathbf{r_1^- = min_{i=1,3}\{ v_{i1} \}; \; r_2^- = min_{i=1,3}\{ v_{i2} \}; r_3^- = max_{i=1,3}\{ v_{i3} \}; r_4^- = max_{i=1,3}\{ v_{i4} \};} \end{aligned} \qquad (6)$$

and so (8)

D^+	0,113	0,200	0,113	0,151
D^-	0,033	0,071	0,318	0,400

(8)

then ,

$$Q^+ = \begin{array}{|c|}\hline 0{,}049 \\ \hline 0{,}276 \\ \hline 0{,}279 \\ \hline \end{array} \text{ şi } Q^- = \begin{array}{|c|}\hline 0{,}342 \\ \hline 0{,}175 \\ \hline 0{,}123 \\ \hline \end{array}$$

and finally (9)

$$S_2 = \begin{array}{|c|}\hline 0{,}875 \\ \hline 0{,}388 \\ \hline 0{,}306 \\ \hline \end{array} \qquad (9)$$

The final classification becomes now $\mathbf{C_1} \succ \mathbf{C_2} \succ \mathbf{C_3}$

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For an opinion poll , different input data occurs . In the following we'll specify the category of data that will be used .

A normed discrete fuzzy set is an object having the next aspect

$$\mathbf{X}=\begin{pmatrix}\mathbf{x}_1 & \mathbf{x}_2 & \dots & \mathbf{x}_i & \dots & \mathbf{x}_n\\ \mathbf{p}_1 & \mathbf{p}_2 & \dots & \mathbf{p}_i & \dots & \mathbf{p}_n\end{pmatrix}$$

subject to the next conventions :

- $\{\mathbf{x}_i\}_{i=\overline{1,n}}$ are the elements of the set X

- $\{\mathbf{p}_i\}_{i=\overline{1,n}}$ are the degrees of appartenence of the set X : so , p_i is the degree to which the element x_i belongs to the set X ; note that this degrees of appartenence satisfy the normality conditions : $\mathbf{p}_i \geq 0, (\forall)\, \mathbf{i}=\overline{1,n}\ ; \sum_{i=1}^{n}\mathbf{p}_i = 1.$

Two fuzzy sets , $\mathbf{X}=\begin{pmatrix}\mathbf{x}_1 & \mathbf{x}_2 & \dots & \mathbf{x}_i & \dots & \mathbf{x}_n\\ \mathbf{p}_1 & \mathbf{p}_2 & \dots & \mathbf{p}_i & \dots & \mathbf{p}_n\end{pmatrix}$ and $\mathbf{Y}=\begin{pmatrix}\mathbf{x}_1 & \mathbf{x}_2 & \dots & \mathbf{x}_i & \dots & \mathbf{x}_n\\ \mathbf{q}_1 & \mathbf{q}_2 & \dots & \mathbf{q}_i & \dots & \mathbf{q}_n\end{pmatrix}$

are equal if $\mathbf{p}_i = \mathbf{q}_i, (\forall)\, \mathbf{i}=\overline{1,n}$ and disjoint , if $\mathbf{p}_i \cdot \mathbf{q}_i = 0, (\forall)\, \mathbf{i}=\overline{1,n}$.

For two normed fuzzy sets X,Y , different similarity coefficients are defined , for example :

$$\mathbf{r}(\mathbf{X},\mathbf{Y})=\frac{\sum_{i=1}^{n}\mathbf{p}_i\cdot\mathbf{q}_i}{\sqrt{\sum_{i=1}^{n}\mathbf{p}_i^2\cdot\sum_{i=1}^{n}\mathbf{q}_i^2}}\ ;\ \mathbf{R}(\mathbf{X},\mathbf{Y})=\frac{2\cdot\sum_{i=1}^{n}\mathbf{p}_i\cdot\mathbf{q}_i}{\sum_{i=1}^{n}\mathbf{p}_i^2+\sum_{i=1}^{n}\mathbf{q}_i^2}\ .$$

Both of this coefficients have the next properties :

- are sub- unitary , non- negative ($0 \leq \mathbf{r}(\mathbf{X},\mathbf{Y}) \leq 1\ ;\ 0 \leq \mathbf{R}(\mathbf{X},\mathbf{Y}) \leq 1$)
- if the coefficient equals zero , then the fuzzy sets are disjoint
- if the coefficient equals 1 , then the fuzzy sets are equal

Let for example $\mathbf{X}=\begin{pmatrix}1 & 2 & 3\\ 0{,}2 & 0{,}3 & 0{,}5\end{pmatrix}, \mathbf{Y}=\begin{pmatrix}2 & 3 & 4\\ 0{,}4 & 0{,}5 & 0{,}1\end{pmatrix}$ be two normed fuzzy sets : let's calculate the similarity coefficients :

first, we have to extend X , Y to get the same set of arguments , i.e.

$$\mathbf{X}=\begin{pmatrix}1 & 2 & 3 & 4\\ 0{,}2 & 0{,}3 & 0{,}5 & 0\end{pmatrix}, \mathbf{Y}=\begin{pmatrix}1 & 2 & 3 & 4\\ 0 & 0{,}4 & 0{,}5 & 0{,}1\end{pmatrix}, \text{ that is ,}$$

$$\mathbf{p}_1=0{,}2;\mathbf{p}_2=0{,}3\,;\mathbf{p}_3=0{,}5;\mathbf{p}_4=0\ ;\mathbf{q}_1=0;\mathbf{q}_2=0{,}4;\mathbf{q}_3=0{,}5;\mathbf{q}_4=0{,}1$$

Finally , $\mathbf{r}(\mathbf{X},\mathbf{Y})=\dfrac{0{,}37}{\sqrt{0{,}38\cdot 0{,}42}}=0{,}926$; $\mathbf{R}(\mathbf{X},\mathbf{Y})=\dfrac{2\cdot 0{,}37}{0{,}38+0{,}42}=0{,}925$

Generally , the values of this two similarity coefficients are close enough .

Note that a common real number , x , corresponds to the fuzzy set $\mathbf{X}=\begin{pmatrix}\mathbf{x}\\ 1\end{pmatrix}$: such an object will be called a crisp number (respectively, a crisp set).

Let’s now return to our problem.

let’s consider the case of an opinion poll : for a crisp data set , a data table with the next shape will be obtained :

TABLE 1:

	criterion 1	criterion 2	…	criterion k
classificable object 1	$\mathbf{r}_{11}$	$\mathbf{r}_{12}$	**…**	$\mathbf{r}_{1k}$
classificable object 2	$\mathbf{r}_{21}$	$\mathbf{r}_{22}$	**…**	$\mathbf{r}_{2k}$
…	**…**	**…**	**…**	**…**
classificable object n	$\mathbf{r}_{n1}$	$\mathbf{r}_{n2}$	**…**	$\mathbf{r}_{nk}$

The values in this table are places in the rankings , according to each criteria : so , $\{ r_{1i} , r_{2i} , \ldots, r_{ni} \}$ is a permutation of $\{ 1 , 2 ,\ldots, n \}$.
This is the case when only one expert is involved
If more than one expert is implicated , the divergent viewpoints can be summarized as in Table 2 below

TABLE 2:

	criterion 1	criterion 2	…	criterion k
classificable object 1	$X_{11} = \begin{pmatrix} 1 & 2 & \dots & n \\ p_{11}^1 & p_{11}^2 & \dots & p_{11}^n \end{pmatrix}$	$X_{12} = \begin{pmatrix} 1 & 2 & \dots & n \\ p_{12}^1 & p_{12}^2 & \dots & p_{12}^n \end{pmatrix}$	…	$X_{1k} = \begin{pmatrix} 1 & 2 & \dots & n \\ p_{1k}^1 & p_{1k}^2 & \dots & p_{1k}^n \end{pmatrix}$
classificable object 2	$X_{21} = \begin{pmatrix} 1 & 2 & \dots & n \\ p_{21}^1 & p_{21}^2 & \dots & p_{21}^n \end{pmatrix}$	$X_{22} = \begin{pmatrix} 1 & 2 & \dots & n \\ p_{22}^1 & p_{22}^2 & \dots & p_{22}^n \end{pmatrix}$	…	$X_{2k} = \begin{pmatrix} 1 & 2 & \dots & n \\ p_{2k}^1 & p_{2k}^2 & \dots & p_{2k}^n \end{pmatrix}$
…	…	…	…	…
classificable object n	$X_{n1} = \begin{pmatrix} 1 & 2 & \dots & n \\ p_{n1}^1 & p_{n1}^2 & \dots & p_{n1}^n \end{pmatrix}$	$X_{n2} = \begin{pmatrix} 1 & 2 & \dots & n \\ p_{n2}^1 & p_{n2}^2 & \dots & p_{n2}^n \end{pmatrix}$	…	$X_{nk} = \begin{pmatrix} 1 & 2 & \dots & n \\ p_{nk}^1 & p_{nk}^2 & \dots & p_{nk}^n \end{pmatrix}$

The samples in Table 2 , namely

$$\begin{pmatrix} 1 & 2 & \dots & n \\ p_{rj}^1 & p_{rj}^2 & \dots & p_{rj}^n \end{pmatrix}$$

are obtained as follows : asking all the experts about the rank of object " r " with respect to the criterion " j " , a percentage p_{rj}^h of them opted to place " h ".

The $\begin{pmatrix} 1 & 2 & \dots & n \\ p_{rj}^1 & p_{rj}^2 & \dots & p_{rj}^n \end{pmatrix}$ **results** are considered to be fuzzy sets ; consequently , two more problems are to be solved : how to determine an unique fuzzy classification set for each criterion , and then , how to determine a finally crisp ranking .

For the first problem , the minimum operator was used ,

$$\tilde{X}_i = \min\{X_{1i}, X_{2i}, \dots, X_{ni}\}, i = \overline{1,k}$$

This option was chosen , for the ranks have the meaning " the smaller , the better ".
If contrary (ie " the greater , the better ") , the " maximum " operator will be prefered.

For he second problem : to establish an unique crisp ranking based on the unique fuzzy one , we'll assign to the objects ranks according to the magnitude of simillarity coefficient between $\tilde{X}_i$ and X_{ij} , for every $j = \overline{1,k}$.

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